Ex: 8.4

Q1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125} = \frac{13}{5^5} \times \frac{2}{2} = \frac{26}{5^5 \times 2} = 0.00416 \Rightarrow$ Terminating decimal expansion

D'r is in form of $2^m \times 5^n$ so

$$\begin{array}{r|l} 5 & 3125 \\ 5 & 625 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$\therefore 3125 = 5^5$

(ii) $\frac{17}{8} = \frac{17}{2^3} \times \frac{5}{5} = \frac{85}{2^3 \times 5} = 1.375 \Rightarrow$ Terminating decimal expansion

D'r is in form of $2^m \times 5^n$ so

$$\begin{array}{r|l} 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

$\therefore 8 = 2^3$

(iii) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

$\hookrightarrow$ D'r $\neq 2^m \times 5^n$ $\therefore$ Non-terminating repeating expansion

$$\begin{array}{r|l} 5 & 455 \\ \hline 7 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

(iv) $\frac{15}{1600} = \frac{15}{2^6 \times 5^2}$

$\hookrightarrow$ D'r $= 2^m \times 5^n$ $\therefore$ terminating decimal expansion

$$\begin{array}{r|l} 2 & 1600 \\ \hline 2 & 800 \\ \hline 2 & 400 \\ \hline 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

(v) $\frac{29}{343} = \frac{29}{7^3}$

$\hookrightarrow$ D'r $\neq 2^m \times 5^n$ $\therefore$ Non-terminating repeating decimal expansion.

$$\begin{array}{r|l} 7 & 343 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

(vi) $\frac{23}{2^3 5^2}$ $\rightarrow$ D'r = $2^m \times 5^n$ $\therefore$ terminating decimal expansion

(vii) $\frac{129}{2^2 5^7 7^5}$ $\rightarrow$ D'r $\neq 2^m 5^n$ $\therefore$ Non terminating repeating decimal expansion

(viii) $\frac{6}{15}$ $\rightarrow$ D'r $\neq 2^m \times 5^n$ $\therefore$ Non terminating repeating decimal expansion

(ix) $\frac{35}{50} = \frac{35}{5^2 \times 2}$

$\hookrightarrow$ $D'r = 2^m \times 5^n$ $\therefore$ terminating decimal expansion

$$\begin{array}{r|l} 5 & 50 \\ 5 & 10 \\ 2 & 2 \\ & 1 \end{array}$$

(x) $\frac{77}{210} = \frac{77}{2 \times 5 \times 7 \times 3}$

$\hookrightarrow$ $D'r \neq 2^n \times 5^n$ $\therefore$ Non terminating repeating decimal expansion.

$$\begin{array}{r|l} 2 & 210 \\ 5 & 105 \\ 7 & 21 \\ 3 & 3 \\ & 1 \end{array}$$

Ex: 8.4

Q2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

(i) $\frac{13}{3125} = \frac{13}{5^5} \times \frac{2}{2} = \frac{26}{5^5 \times 2} = 0.00416 \Rightarrow$ Terminating decimal expansion

D'r is in form of $2^m \times 5^n$ so

$$\begin{array}{r|l} 5 & 3125 \\ 5 & 625 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$\therefore 3125 = 5^5$

(ii) $\frac{17}{8} = \frac{17}{2^3} \times \frac{5}{5} = \frac{85}{2^3 \times 5} = 1.375 \Rightarrow$ Terminating decimal expansion

D'r is in form of $2^m \times 5^n$ so

$$\begin{array}{r|l} 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

$\therefore 8 = 2^3$

(iii) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

$\hookrightarrow D'r \neq 2^m \times 5^n$ $\therefore$ Non-terminating repeating expansion

$$\begin{array}{r|l} 5 & 455 \\ 7 & 91 \\ 13 & 13 \\ & 1 \end{array}$$

(iv) $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = 0.009375$

$\hookrightarrow D'r = 2^m \times 5^n$ $\therefore$ terminating decimal expansion

$$\begin{array}{r|l} 2 & 1600 \\ 2 & 800 \\ 2 & 400 \\ 2 & 200 \\ 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

(v) $\frac{29}{343} = \frac{29}{7^3}$

$\hookrightarrow D'r \neq 2^m \times 5^n$ $\therefore$ Non-terminating repeating decimal expansion.

$$\begin{array}{r|l} 7 & 343 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

(vi) $\frac{23}{2^3 5^2} = 0.115$

$\rightarrow$ D'r $= 2^m \times 5^n$ $\therefore$ terminating decimal expansion

(vii) $\frac{129}{2^2 5^7 7^5}$ $\rightarrow$ D'r $\neq 2^m 5^n$ $\therefore$ Non terminating repeating decimal expansion

(viii) $\frac{6}{15}$ $\rightarrow$ D'r $\neq 2^m \times 5^n$ $\therefore$ Non terminating repeating decimal expansion

(ix) $\frac{35}{50} = \frac{35}{5^2 \times 2} = 0.7$

$\hookrightarrow$ D'r $= 2^m \times 5^n$ $\therefore$ terminating decimal expansion

$$\begin{array}{r|l} 5 & 50 \\ 5 & 10 \\ 2 & 2 \\ & 1 \end{array}$$

(x) $\frac{77}{210} = \frac{77}{2 \times 5 \times 7 \times 3}$

$\hookrightarrow$ D'r $\neq 2^n \times 5^n$ $\therefore$ Non terminating repeating decimal expansion.

$$\begin{array}{r|l} 2 & 210 \\ 5 & 105 \\ 7 & 21 \\ 3 & 3 \\ & 1 \end{array}$$

Ex: 8.4

Q3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form , p/q what can you say about the prime factors of q?

(i) 43.123456789

$$43.123456789 = \frac{43123456789}{1000000000}$$

$$\frac{43123456789}{1000000000} = \frac{43123456789}{10^9}$$

$$\frac{43123456789}{1000000000} = \frac{43123456789}{(5 \times 2)^9}$$

$$\frac{43123456789}{1000000000} = \frac{43123456789}{5^9 \times 2^9}$$

This is in the form of p/q, and the prime factors of q are in terms of 2^m and 5^n.

Hence, the given real number is a rational number.

Ex: 8.4

Q3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form , p q what can you say about the prime factors of q?

(ii) 0.120120012000120000. . .

0.120120012000120000.... is non-terminating and non-repeating and recurring.

It is irrational. Hence cannot be expressed in the form of p/q

Ex: 8.4

Q3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(iii) $43.\overline{123456789}$

The given number is non-terminating but repeating. So, it would be rational.

As the decimal expansion is recurring, the denominator will have prime factors other than 2 and 5.